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NONSTATIONARY-FREQUENCY MEASUREMENTS IN RESEARCHING THERMOPHYSICAL PARAMETERS AND PHENOMENA. PART 1. STEPWISE METHODS

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UDC 536.24

A new approach is illustrated to the theory of thermal conductivity and the use of solutions there, which is a natural consequence of using new nonstationary-frequency methods in experiments.

New industrial processes and process acceleration have made it necessary to produce new substances and materials with preset properties. Correspondingly, there is an increased need for information on the properties of new materials over wide ranges in the state parameters, and measurements of improved accuracy are required.

This has led to extensive researches on the characteristics of gases, liquids, and solids; much of such research in power engineering is concerned with thermophysical parameters, particularly thermal conductivity, specific heat, thermal expansion, and emissivity. The changes in the level and volume of research have naturally led to considerable attention being given to methods, particularly design and implementation of new fast but precise methods of measuring thermophysical parameters. This has led to radical reequipment in experimental thermophysics and to the replacement of cumbersome and slow classical methods by modern fast and highly informative measurement suites. The major trends in experimental techniques here include not only the general one of improving the informativeness but also the following: devising, developing, and implementing methods for the simultaneous determination of several thermophysical parameters, the general use of electronic methods and apparatus, measurement automation (including computerized data acquisition, control, and data processing), the use of nondestructive and contactless methods to provide high throughput in all measurements, including specimen installation and changing, substantial reductions in error levels, improved metrological systems, the design of essentially new methods for indirect measurements based on techniques in molecular optics and acoustics, and substantial extensions to the temperature and pressure ranges covered.

There are two main groups of methods meeting these requirements: a) pulse and stepwise (monotone) heating, and b) periodic heating. These provide a basis for robotic monitoring and control systems for thermophysical parameters of materials and structures in various industrial processes.

The resolving power and response rate are improved if these methods are used with phase and frequency techniques rather than amplitude ones.

From the information-theory viewpoint, frequency methods are the most informative, and these are increasingly used. However, they are usually based on resonance, where the accuracy is governed mainly by the quality factor for the resonance sensor, not by the accuracy in frequency measurement, so the metrological characteristics of frequency measurements cannot be utilized fully.

This disadvantage is overcome in the new nonstationary frequency methods [1, 2], whose distinctive feature is that the frequency shift in the output signal from the primary sensor provides information on the rate of change in some physical parameter, and if that parameter is the temperature, one measures the rate of change as a function of time and coordinates.

The transfer from temperature measurement to rate measurement involves a new methodology in formulating boundary-value problems and in using the solutions.

It is often not necessary to known the temperature pattern in a nonstationary treatment; one can then formulate the problem in a way such that the desired pattern is that of the rate of change in temperature, not the temperature itself. If it is necessary to transfer to the temperature pattern, it is sufficient to integrate the solution with respect to time.

As an illustration, we consider a simple problem, which we formulate first for the temperature pattern and then for the rate of change.

For a semiinfinite body [3]

$$\frac{\partial T(x, \tau)}{\partial \tau} = a \frac{\partial^2 T(x, \tau)}{\partial x^2} \quad (\tau > 0, \quad 0 < x < \infty),$$

$$T(x, 0) = T_0, \quad T(0, \tau) = T_c, \quad \frac{\partial T(+\infty, \tau)}{\partial x} = 0.$$

We solve this by the operational method to get

$$\frac{T(x, \tau) - T_c}{T_0 - T_c} = \mathrm{erf}\left(\frac{x}{2\sqrt{a\tau}}\right) \tag{1}$$

We formulate the problem for the rates of change:

$$\frac{\partial V(x, \tau)}{\partial \tau} = a \frac{\partial^2 V(x, \tau)}{\partial x^2}, \quad V(x, 0) = 0,$$

$$V(0, \tau) = (T_c - T_0)\delta(\tau), \quad \frac{\partial V(+\infty, \tau)}{\partial x} = 0.$$

We solve the problem by the operational method to get

$$V(x, \tau) = \frac{\Delta T_0 x}{2\sqrt{a\pi\tau^3}} \exp\left(-\frac{x^2}{4a\tau}\right). \tag{2}$$

On differentiating (1), we get the same result.

The two methods are equivalent in complexity, at least as regards this class of problem; if the treatment is facilitated on reformulation in terms of rates of change, the return to the solution for the temperatures balances out the advantage. Nevertheless, the boundary and initial conditions are often simplified. For example, if there is an initial temperature distribution as a function of the coordinates, we have a zero initial condition in the new formulation. The advantages may be felt in solving much more complicated problems, particularly if it is not necessary to return to the solution for the temperature.

The advantages of our method are fully observed for inverse treatments. There are many additional complications here, as for inverse problems generally. Often, such a problem violates the conditions for uniqueness and stability, so regularization methods are required.

If a thermal process is not very fast and the sensors are fairly close to the surface, while precision measurements are made, one can use direct methods in the inverse treatments, i.e., the inverse problem is solved in the initial formulation but by means of approximate methods that allow one to check the measure of the closeness between the solution and the exact one by varying the usual parameters in the computational algorithms (argument steps, number of terms in series, number of iterations, etc.).

One usually employs a nonlinear thermal-conduction equation in an inverse treatment, and a study has been made [4] of the errors arising from using a linear equation instead of a nonlinear one.

However, $\lambda(T)$, $c(T)$, and $a(t)$ can be taken as independent of temperature for certain materials and not very wide temperature ranges, so one can use the linear equation written for the temperature or rate pattern. Many methods have been devised for determining constant λ, c, and a, both stationary and nonstationary. Nonstationary (pulse and step or periodic) ones meet current requirements best.

We restrict consideration to one of the step methods.

We rewrite the solution to (1) as

$$\Delta T(\text{Fo}) = \Delta T_0 \text{erf} \left(\frac{1}{2\sqrt{\text{Fo}}} \right),$$ (3)

where

$$\text{Fo} = \frac{a\tau}{x^2}.$$

The structure does not allow one to derive an analytical formula for the thermal diffusivity; one determines a by recording $\Delta T(\text{Fo})$ at a given point x in a semiinfinite body and calculates the relative excess temperature $\Delta T(\text{Fo})/\Delta T_0$, tables then being used to derive $\text{erf}(1/2\sqrt{\text{Fo}})$ for the corresponding Fo. The thermal diffusivity is

$$a = \frac{x^2}{\tau} \text{Fo}.$$

We now consider the solution:

$$\frac{\partial \Delta T(x, \tau)}{\partial \tau} = V(x, \tau) = \frac{\Delta T_0 x}{2\sqrt{a\pi\tau^3}} \exp\left(-\frac{x^2}{4a\tau} \right).$$ (4)

One can use numerical or approximate analytical methods to solve this transcendental equation.

We take another approach and examine the behavior of $\Delta T(x, \tau)$, $V(x, \tau)$, $\partial/\partial\tau V(x, \tau)$.

The point of inflection A (Fig. 1a) corresponds to the maximum in $V(x, \tau)$, while the points of inflection B and C (Fig. 1b) correspond to turning-point values in $V'(x, \tau)$, namely points D and E in Fig. 1c.

It is complicated to detect the point of inflection on the $\Delta T(x \tau)$ graph, while if one measures $V(x, \tau)$ rather than $\Delta T(x, \tau)$, one has an obvious prominent peak M, which corresponds to time τ_0. The maximum and the argument corresponding to it are readily observed, so they can be used to identify a.

We differentiate (4) and equate it to 0:

$$\frac{\partial V}{\partial \tau} = -\frac{\alpha}{\tau_{\max}^{5/2}} \left(\frac{k}{\tau_{\max}} + \frac{3}{2} \right) = 0, \text{ where } k = -\frac{x^2}{4a}, \quad \alpha = \frac{\Delta T_0 x}{2\sqrt{\alpha\pi}}, \quad \frac{k}{\tau_{\max}} = -\frac{3}{2};$$

if $\tau_{\max}$ is known, then $k = -(3/2)\tau_{\max}$, so

$$a = \frac{x^2}{6\tau_{\max}}.$$ (5)

If it easier to record $V(\tau_{\max})$, we can substitute $\tau_{\max} = -(2/3)k$ into (4) and get

$$a = \frac{1}{6} \sqrt{\frac{2\pi}{3} \frac{V(\tau_{\max}) x^2}{\Delta T_0 \exp\left(-\frac{3}{2} \right)}},$$ (6)

expressed in terms of $V(\tau_{\max})$.

This method enables one to determine λ and c if one knows the heat flux absorbed by the body in a certain time.

According to (3), the amount of heat given up (absorbed) by a probe in time $d\tau$ over a section S is [3]

$$dQ = -\lambda S \left(\frac{\partial T}{\partial x} \right)_{x=0} d\tau = -\lambda S \frac{\Delta T_0}{\sqrt{\pi a\tau}} d\tau,$$ (7)
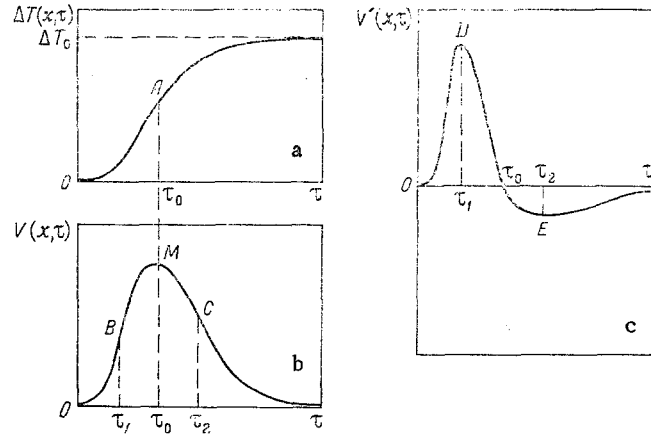
Fig. 1. Time dependence of the temperature $\Delta T(x, \tau)$ and of the first derivative $V(x, \tau)$ and the second derivative $V'(x, \tau)$ with respect to time (parts a, b, and c respectively) at any point x.

and in the time interval $\tau_2 - \tau_1$

$$Q = \frac{2Sb(T_c - T_0)(\sqrt{\tau_2} - \sqrt{\tau_1})}{\sqrt{\pi}} . \tag{8}$$

The latter gives the thermal activity coefficient $b = \sqrt{\lambda c}$, which enables one to determine λ and c if $a = \lambda/c$ is known.

If one can measure $\partial Q(0, \tau)/\partial \tau$, one can use (4) and (7) to get

$$\frac{\partial Q(0, \tau_2)}{\partial \tau} - \frac{\partial Q(0, \tau_1)}{\partial \tau} = \frac{bS\Delta T_0}{\sqrt{\pi}} \frac{\sqrt{\tau_1} - \sqrt{\tau_2}}{\sqrt{\tau_1 \tau_2}} , \tag{9}$$

from which one finds b and the other coefficients.

There is another simple relation that reduces the measurement time substantially. We assume that we know

$$V_1 = \alpha \tau_1^{-3/2} \exp\left(-\frac{x^2}{4a\tau_1}\right) \text{ and } V_2 = \alpha \tau_2^{-3/2} \exp\left(-\frac{x^2}{4a\tau_2}\right) .$$

We divide one equation by the other and express a as

$$a = \frac{x^2 \Delta \tau}{4\tau_1 \tau_2} \left(\ln \frac{V_2}{V_1} \left(\frac{\tau_2}{\tau_1} \right)^{3/2} \right)^{-1} . \tag{10}$$

If $\Delta V/V \ll 1$ and $\Delta \tau/\tau \ll 1$, this exact formula becomes the approximation

$$a = \frac{x^2}{4\tau_1} \left(\frac{\Delta V}{\Delta \tau} \frac{\tau_1}{V_1} + \frac{3}{2} \right)^{-1} . \tag{10a}$$

If we take $(\partial V/\partial \tau)_{\tau_1}$ instead of $\partial V/\Delta \tau$, (10a) becomes the exact result

$$a = \frac{x^2}{4\tau} \left(\frac{\partial V}{\partial \tau} \frac{\tau}{V} + \frac{3}{2} \right)^{-1} . \tag{10b}$$

As $\partial V/\partial \tau$ is not measured directly, it is complicated to use (10b), so we restrict consideration to (10).

We estimate the error in measuring the quantity

$$\left| \frac{\delta a}{a} \right| = 2 \left| \frac{\delta x}{x} \right| + \left| \frac{\delta \Delta \tau}{\Delta \tau} \right| + \left| \frac{\delta \tau_1}{\tau_1} \right| + \left| \frac{\delta \tau_2}{\tau_2} \right| + \left| \frac{\delta \ln \frac{V_2}{V_1} \left(\frac{\tau_2}{\tau_1} \right)^{3/2}}{\ln \frac{V_2}{V_1} \left(\frac{\tau_2}{\tau_1} \right)^{3/2}} \right| .$$

As the time measurement accuracy is higher by several orders of magnitude than the accuracy in measuring the other quantities, the time terms can be neglected:

$$\left| \frac{\delta a}{a} \right| = 2 \left| \frac{\delta x}{x} \right| + \left| \frac{\delta V_1}{V_1} \right| \frac{\left| 1 + \left| \frac{V_1}{V_2} \right| \right|}{\left| \ln \frac{V_1}{V_2} \right|} . \tag{10c}$$

here we have used the obvious inequality

$$\left| \ln \frac{V_2}{V_1} \left(\frac{\tau_2}{\tau_1} \right)^{3/2} \right|^{-1} \leqslant \left| \ln \frac{V_2}{V_1} \right|^{-1}.$$

We impose the condition

$$\frac{\left| + \left| \dfrac{V_1}{V_2} \right| \right|}{\left| \ln \dfrac{V_1}{V_2} \right|} \leqslant 1.$$

We solve this transcendental equation to get

$$\left| \frac{V_2}{V_1} \right| \geqslant 3{,}57. \qquad (10\text{d})$$

If this condition is met, then

$$\left| \frac{\delta a}{a} \right| = 2 \left| \frac{\delta x}{x} \right| + \left| \frac{\delta V_1}{V_1} \right|. \qquad (10\text{e})$$

This shows that the error in determining a is governed by δx and δV_1 and by the values chosen for x and V_1. We specify $|\delta a/a|$ and use the known δx and δV_1 to arrive at lower limits for x and V_1. If we know the order of a, we can use the proposed x and ΔT_0 to estimate the maximum rate of change at a given point x:

$$V_{\max} = \sqrt{\frac{54}{\pi} \frac{\Delta T_0 a}{x^2} \exp\left(-\frac{3}{2} \right)}.$$

In choosing V_1, one must remember that V_1 is less than $V_{\max}$; to estimate the lag in the method, we give detailed values.

Let $\Delta T_0 = 1°K$, $x = 10^{-3}$ m, $a = 10^{-7}$ m^2/sec, and $\delta x = 10^{-4}$ m, $\delta V = 10^{-3}$ K/sec. The order of a corresponds to insulating materials. The order of δV is set from the actual performance of nonstationary-frequency methods. With $|\delta a/a| \sim 10^{-1}$ we get that $\tau_{\max} \approx 2$ sec. The lower limit to V_1 is 10^{-2} K/sec, which corresponds to a time of 0.4 sec. During this time, the temperature at point x increases by only $1.5 \cdot 10^{-3}$ K, which corresponds to the error of measurement in nonstationary-frequency methods. If we wish to use such methods, the temperature can begin to be measured (on the basis of $|\delta a/a|$) only from $\tau = 1$ sec, which is five times later than the instant when rate measurement starts. If on the other hand one uses traditional amplitude methods to measure ΔT (for example, by means of thermocouples), the time for starting the measurements will be larger by several orders of magnitude.

Then transferring from measuring $T(x, \tau)$ to $V = \partial T(x, \tau)/\partial \tau$ provides simple working formulas and enables one to evaluate the performance of a step method.

This comparison has been made for step methods on a linear model, and the numerical estimates apply insofar as the model is correct. When one uses nonstationary methods, it is very important in estimating the error to consider the lag in the sensors. These aspects and some others require separate consideration.

NOTATION

T, temperature; V, rate of temperature variation; x, space coordinate; τ, time; a, thermal diffusivity; $\delta(\tau)$, Dirac delta function; ΔT, difference between the temperature at point x at time $\tau > 0$ and the temperature at the boundary at $\tau = 0$; ΔT_0, difference between the temperature at the boundary at time $\tau > 0$ and temperature at the initial time; λ, thermal conductivity; c, heat capacity; Fo, Fourier number; A, amount of heat; S, cross-sectional area; b, thermal activity coefficient.

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